

Notes on Entropic Interpretation of Gravity

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Abstract

Some essential conceptual aspects that will fill some logical gaps of the frame to interpret the gravity as an entropic force was investigated, we focus on some crucial issues that didn't emphasized in Verlinde's original paper[arXiv:1001.0785]. This note explains the context that holographic screen can be endowed with an entropy that proportional to it's area and the meaning in using equipartition law in spacetime thermodynamics, thermodynamic quantities such as entropy and temperature are observer dependent is the crucial concept in explaining those problems. Coarse graining will leave information in the gravitational potential, which will connect different observer's point of views for the same object. This will help us to understand the coarse graining dependent definition of entropy and the nature of spacetime. It also indicates the way that entropy bounds work, which is consistant with Bekenstein entropy bound and holographic entropy bound.

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I. INTRODUCTION

Since the discovery of black hole thermodynamics by Bekenstein[1] and Hawking[2], the concept that gravity is an emergent phenomenon has been widely approved. This idea that there should be internal degrees of freedom exist for spacetime can be traced back to the year 1968, when Sakharov found some similar behavior in the elasticity and the spacetime dynamics[3]. The similarity between gravitational force and Elastic force is not just a coincidence, but have some profound connections. Elastic force is not seem as a fundamental force, gravitational force should also not fundamental, if elasticity and gravity originate from the same kind of mechanism. Current gravity theory such as Einstein's general relativity may be an emergent low-energy long-distance phenomenon that is insensitive to the details of the underlying quantum theory of gravity. It will be very exciting if we have an statistical description of the internal degrees of freedom of spacetime, although this is just an extravagant hope at present.

In 2010, a paper by Erik P. Verlinde attracts tremendous attention. He advocated the idea that gravity is an emergent phenomenon and asserted that gravity is an entropic force in [4]. The main idea of entropic interpretation of gravity is summarized as follows: gravitational system have an microscopic structure, that is to say, it is a thermodynamic system that all of the thermodynamic variables can be endowed, gravity is just a statistical tendency to return to a maximal entropy state. Based on some general assumptions, such as the holographic principle, the equipartition law of energy and the Unruh temperature formula, the Newton's law of gravity and the second law of Newton can be derived.

The entropic interpretation of gravity give rise to a variety of debates, the scientific community is divided into two sides, some people support the idea, others don't, for the criticism, see [5] and [6]. The entropic interpretation of gravity give us a new perspective in the study of cosmology, the Friedmann equation can be derived by using this frame[7, 8]. It also have been used to study the dark energy[9, 10], cosmological inflation and acceleration[11, 12].

In this note, we disscuss some essential conceptual aspects of this paradigm that is still obscure in Verlinde's paper. As we have stated in [13], those aspects stated below should be further investigated:

1. In Verlinde's paper [4], the boundary of a gravitational system is a suppositional holographic screen, the derivation of the dynamic equation of gravity and the discussion

of the emergence of spacetime is all related to the holographic screen. However, there is no explicit explanation in [4] that how can we endow the holographic screen a temperature and an entropy. It should be noticed that the temperature and entropy of a gravitational system is observer dependent, since we can only endow a temperature and an entropy to an observer dependent horizon, when it comes to a holographic screen, we need an explanation to fulfill the logical gap.

2. The equipartition law is a key assumption in deriving Newton's law and Einstein's field equation in [4], but we still don't know the implication of being used to spacetime thermodynamics and the context of its application.
3. In [4], holographic screen is viewed as the boundary that separates the spacetime into an emergent part and a non-existent part. It is of no sense to separate the space into two regions with a suppositional screen, and said that one part exists and the other is not. What is the real edge that separates the emergent spacetime region from which has not emerged yet?

Those problems stated above are crucial, we want to clarify those problems in this note. It is surprising that they are all closely related to the properties of horizon, and thermodynamic quantities are observer dependent in the thermodynamic description of spacetime. All those problems will be automatically resolved after we made some conceptual change about the spacetime.

Some authors have developed a formal thermodynamic first law on holographic screens with spherical symmetry [14, 15], but the physical implication of their results is ambiguous. We found that our result will provide an explanation to what is the explicit physical meaning of the thermodynamic parameters used in their formula.

This note will be organized as follows: In sec.II, the important role of horizon played in spacetime thermodynamics was discussed, and we explain the context that holographic screen can be endowed with an entropy that is proportional to its area. In sec.III, we illustrate the meaning and conditions when we apply the thermodynamic equipartition law to spacetime thermodynamics. In sec.IV, we discuss coarse graining in gravitational system and its implication in understanding the coarse graining dependent definition of entropy and the entropy bounds. Discussions and conclusions were made in sec.V.

II. HORIZONS AND HOLOGRAPHIC SCREENS

In general relativity, light cones are effected by gravity, and it follows that there will exist observers who do not have access to part of the spacetime, that is to say those observers will perceive horizons. It should be noticed that all horizons are observer dependent, and in general relativity all observers are set on an equal footing because this is the crucial reason for Einstein to establish general relativity, we should also treat all horizons equally in the study of spacetime thermodynamics[16, 17].

Compared to ordinary screens in spacetime, horizons have distinctive features. It has a key property that it can block information from the corresponding observer which ordinary screens don't have, it is this property of horizon that make us to endow an entropy to the horizon for the observers who perceive the horizon. Supposing energy flows across the horizon, the entropy in the region accessible to the observer can decrease because the entropy carried by the energy flow is not accessible to the observer any longer, if the horizon doesn't have an entropy, the second law of thermodynamics will be violated. Therefore, horizon should have an entropy and a corresponding temperature, and the field equation of gravity can be derived by demand the Clausius relation is hold on horizons[17, 18].

It seems unnatural to endow an entropy to ordinary screens, because it is not necessary to require ordinary screens have an entropy to avoid the violation of the second law of thermodynamics, the information carried by energy flow across a ordinary screen is still approachable for the corresponding observer. In fact, the distinction between horizons and ordinary screens is artificial, the black hole horizon is just an ordinary screen from the point of view of an free falling observer, and we can construct local Rindler horizons to cover every small patch of any ordinary screens[19], is it a horizon or not is all depend on which kinds of observers watch it.

It should be noticed that observer dependent is a crucial concept in the discussion of following paragraphs. The holographic screen is introduced by susskind to illustrate the holographic principle[20]. For an observer a holographic screen could be a horizon or an ordinary screen that encompass the horizon[21, 22]. The holographic principle[20, 23] states that a macroscopic region of space and everything inside it can be represented by a boundary theory living on the boundary of the region. The strongest supporting evidence for the holographic principle comes from black hole physics and the AdS/CFT correspondence, in

those two cases the number of degrees of freedom of the boundary surface agrees with the number of physical degrees of freedom contained in the bulk. The entropy of a Schwarzschild black hole, $S_{BH} = A_{horizon}/4$, precisely saturates the holographic entropy bound. In this sense, if we view the black hole as an isolate system, a black hole is the most entropic object one can put inside a given spherical surface. This is not surprising since the gravitational evolution can be viewed as a thermodynamic process for a system to reach an equilibrium state that the holographic principle attained[13]. For complete weakly self-gravitating physical system surrounded by a spacelike surface with area A , entropy of the system, $S < A/4$, will not saturate the bound[20], notice that this surface is not a horizon from the point of view of the observers that view the system as an ordinary one. We may take the surface as a holographic screen, and there possibly exists a theory on it –though hard to establish, and may have peculiar properties, don’t like the AdS/CFT– that dual to the bulk theory for the system. The difficulty in establishing such theory probably have something to do with redundant degrees of freedom on holographic screen, we will discuss it in sec.IV.

For a gravitating system enclosed by a holographic screen, we can construct local Rindler horizons for every observers placed on the holographic screen who will experience an acceleration a produced by the gravitational body, then, we can attribute an Unruh temperature $T = \frac{\hbar a}{2\pi k_B c}$ to it, and attribute an entropy

$$S_{screen} = \frac{c^3}{4G\hbar} \int_S dA \quad (1)$$

to the holographic screen. Obviously, this amount of entropy will violate the Bekenstein entropy bound[4, 24], we will clarify this problem in sec.III and IV. One can think about the holographic screen as a storage device for information, as stated in[4], we call the fundamental degree of freedom (or fundamental atom) on holographic screen bit, note that the bit here is not the unit used to measure information. If we assume the holographic principle holds, the total number of bits N on holographic screen is proportional to its area A , that is

$$N = \frac{Ac^3}{G\hbar}. \quad (2)$$

The degrees of freedom is consistent with entropy formula eq.1, and the dynamics of those bits on holographic screen is governed by the unknown dual theory mentioned above, which also rules how to store information by bits. Note that, for an observer, the holographic principle doesn’t set the horizon and ordinary screen on an equal footing. The maximal

storage capability of a holographic screen equals the total number of bits N when the holographic screen is a horizon, that is encode one bit information by one fundamental degree of freedom, which happens when the holographic entropy bound is saturated. When the holographic entropy bound is not saturated and the holographic screen is not a horizon, more than one bits are used to encode one bit information, which is stored and processed in an unknown coarse graining way.

III. THE OBSERVER DEPENDENCE OF EQUIPARTITION LAW

Horizon can block information from the corresponding observer and separate the space-time into two different parts, one part is accessible and the other is not. In this sense, the horizon plays as a boundary of a gravitational system, the system is not separated from the other by a suppositional screen, but by a causality barrier, and the “system” contain the degrees of freedom beyond the horizon which can have a dual description on the horizon.

A complete understand of the fundamental degree of freedom requires a consistent theory of quantum gravity, which has so far proved elusive. However, just as semiclassical analysis such as the Bohr model was important in the early development of quantum mechanics, a similar approach may be helpful in understanding some of the microscopic features of spacetime. To gain some intuitive understanding of the spacetime atom, let us do some semiclassical analysis for a Schwarzschild black hole with Schwarzschild radius R and mass M . The number of the microscopic degrees of freedom of the black hole is $N = A_{horizon}$. If we divide the black hole mass M evenly over the microscopic degrees of freedom, each spacetime atom will have an energy $E_\gamma = \frac{1}{8\pi R}$, and the Compton wavelength of it is $\lambda_\gamma \sim R$. This means the black hole horizon behaves like a “box” that confine the whole spacetime atom within it. Modes with Compton wavelength $\gtrsim R$ can not be confined in the black hole, and do not contribute to the entropy of the black hole; while modes with Compton wavelength $\lesssim R$ is not the most effective way to increase the entropy of the black hole, this is conflict with that black hole is a most entropic object. We speculate that the formation of a black hole is to distribute ordinary form of matter in a manipulative way with efficiency to make it as a most entropic object from the point of view of a proper observer. There is a gap in magnitude between black hole entropy and ordinary entropy, it was proven in [25] that entropy bound for ordinary system is bounded by $A^{\frac{3}{4}}$, it should undergo a peculiar

process when the black hole is formed, we will come back to this issue in sec.IV.

The thermodynamic description of spacetime is independent of the exact nature of the degrees of freedom, although we don't have definite knowledge about atoms of spacetime, we can apply the thermodynamic laws to gravitational system. In Verlinde's original paper [4], the equipartition law play a crucial role in deriving Newton's law and Einstein's field equation. This is analogous to what we did to a Schwarzschild black hole, the total energy E for a system is divided evenly over the microscopic degrees of freedom of spacetime N :

$$E = \frac{1}{2} N k_B T \quad (3)$$

where k_B is Boltzman's constant and T represents the temperature of the system. We can get this result if we attribute an energy $(1/2)k_B T$ to each microscopic degree of freedom of spacetime.

In Verlinde's paper [4], the meaning of the temperature T is obscure. When T equals the Unruh temperature $T = \frac{\hbar a}{2\pi k_B c}$, we can get the second law of Newton: $F = ma$; if T is explained as the temperature of the holographic screen and also equals the Unruh temperature, the following entropy formula (1) of the screen will violate the Bekenstein entropy bound as we stated in eq.(1). What are the reasons of this discrepancy ? In fact, this discrepancy could be avoid after some issues of this problem are clarified. The entropy of a complete weakly self-gravitating physical system is bounded by the Bekenstein entropy bound[26], and the Bekenstein entropy bound has been explicitly shown to hold in wide classes of equilibrium systems[27], the system considered in Verlinde's paper is unlikely to violate the Bekenstein entropy bound. On the other hand, it is absurd to conclude that the temperature of the holographic screen is not the Unruh temperature, although the Unruh temperature formula is not necessary in deriving Newton's law of gravitation $F = G \frac{Mm}{R^2}$, if this is true, the gravitational force should not an ordinary force which obey the second law of Newton, since the origin of the temperature are different.

We should note that the temperature and entropy are observer dependent quantities in the thermodynamical description of gravity[28]. An observer falling into a black hole will ascribe different thermodynamic properties to the black hole compared to an observer who is remaining stationary outside the horizon. It is also true that an inertial observer will attribute different temperature and entropy to the Minkowski vacuum compared to a Rindler observer. The entropy of the holographic screen $S_{screen} = \frac{c^3}{4G\hbar} \int_S dA$ is associate

with the observers who will experience an Unruh temperature $T = \frac{\hbar a}{2\pi k_B c}$, those observers will ascribe this entropy to the holographic screen because they experience an acceleration produced by the gravitational body and we can construct local Rindler horizon for any small patch of the holographic screen, any other observers will not attribute the same amount of entropy to the holographic screen.

When one does quantum field theory in curved spacetime, “particle” become an observer dependent notion, it is not surprise to associate different amount of entropy with a gravitational system for different kinds of observers. In [29], the authors showed that the entropy associated with a ordinary localized object in flat and otherwise empty space is not an invariant quantity defined by the system alone, but rather depends on which observer we ask to measure it. From the inertial observer’s point of view, the entropy of an object with n possible microstates and energy δE is $\delta S_{inertial} = \ln n$, from the point of view of the Rindler observer, the entropy of the object with the same resolution is

$$\delta S_{Rindler} = \frac{\delta E}{T}, \quad (4)$$

we see the Rindler entropy is not necessary equals to the inertial entropy, the relation between those two kinds of entropy is still in the dark, but we have reasonable ground to believe this problem will be uncovered after we understand the nature of spacetime, in section IV, we will argue that it has something to do with coarse graining dependent of entropy. From the Rindler observer’s point of view the relation (4) can be interpreted as the first law of thermodynamics on holographic screens in some sense[14], if an object carries energy δE falling into the local Rindler horizon, the associated change of the holographic screen entropy will be $\frac{\delta E}{T}$.

Now, the obscureness in apply the equipartition law to gravitational system discussed above can be cleared up. When we want to measure the length of a object, we need a ruler, which means we have to use certain quantity of length as the standard length, similarly, we can apply the equipartition law to a gravitational system because spacetime have microscopic degrees of freedom. We have to conclude that the application for the equipartition law is also observer dependent, since the fundamental energy $k_B T/2$ is temperature dependent, and different T correspond to different Rindler observer. We should note that in Schwarzschild spacetime a infinity observer is immersed in a Hawking radiation with temperature $T = \frac{1}{8\pi M} = \frac{1}{4\pi R}$, this observer will have a “ruler” with standard fundamental

energy $\frac{1}{2}k_B T = \frac{k_B}{8\pi R}$, it happens that each fundamental degree of freedom have the same energy as we discussed above. In the above case used in Verlinde's paper, Rindler observers in different spherical surfaces will experience different Unruh temperatures, and will have different "ruler", so they will divide the same system into different amount of degrees of freedom. Eq.(3) can be thought as the integrated form of eq.(4)[14].

IV. COARSE GRAINING OF ENTROPY AND ENTROPY BOUND

We want to point out that we can't talk about the entropy of a holographic screen without regarding to the circumstance of physical system. For a inertial observer, it is ridiculous to endow an entropy to a screen in Minkowski spacetime that is equal to it's area, similarly, for a observer placed at infinity, the entropy of a Schwarzschild black hole will not change when we go away from the horizon and choose a holographic screen that contains the horizon inside it, the observer dependent entropy is always equals to $\frac{A_{horizon}}{4}$. We therefore conclude that information can be stored on screens, and the amount of information that stored on it is determined by circumstance of the whole system and the corresponding observer that measures it, rather than the screen itself. Generally speaking, the capacity of the screen to store information is no less than information stored on the whole system surrounded by it unless the holographic screen happened to be the horizon of the corresponding observer.

Entropy is an extremely subtle concept in general relativity, a proper framework for general discussion of entropy is still lack[30]. We have known that entropy is an observer dependent quantity and it's definition is coarse graining dependent, a question follows, what is the relationship between observers and coarse graining? We will see that the entropic force interpretation of gravity will give us some clue about this problem.

Verlinde's entropic force interpretation of gravitational force is really of some attractive properties and profound meanings. It is the first time that uncovered the origin of the gravitational force and inertia, and the mechanism of gravity is really clear and imaginable. Lots of interesting application of Verlinde's proposal have been studied, in [9], the authors showed that the UV/IR relation proposed by Cohen et al., as well as holographic dark energy can be derived from the entropic force formalism. The crucial equation used in [9] is

a relation between entropy S , used bits N and Newton potential Φ :

$$\frac{S}{N} = -k_B \frac{\Phi}{2c^2}. \quad (5)$$

The Newton potential Φ can be identified as a coarse graining variable, and $0 \leq -\frac{\Phi}{2c^2} \leq \frac{1}{4}$. This relation is extremely consistent with our illustration in section II, because it makes it clear that the number of bits on the holographic screen which are used to dually describe the object in the bulk can be either equal to or larger than the entropy of the bulk object. Consider two observers placed on two different equipotential holographic screens with Newton potential Φ_1 and Φ_2 , they will experience different acceleration produced by the bulk object, and will endow two observer dependent entropy S_1 and S_2 to the gravitating object, according to eq.(5), S_1 and S_2 satisfy

$$\frac{S_1}{S_2} = \frac{\Phi_2}{\Phi_1}, \quad (6)$$

we see, this relation indicate a relationship between observers and coarse graining, in other words, Newton potential Φ is a phenomenological parameter, which keeps track of the message for a coarse graining description of the bulk object on different holographic screens. Observers at rest on different holographic screens will experience different acceleration produced by the bulk object, different observers will endow different entropy to the bulk object, as we discussed in section III, and those different entropy correspond to different observers are link by gravitational potential.

We can use another example to illustrate gravitational potential measures the amount of coarse graining. Consider a object with proper energy E that can surrounded by a sphere with radius r in a Schwarzschild black hole background, with Schwarzschild radius R and mass M . When place the object at infinity, a infinity observer is immersed in a Hawking radiation with temperature $T = \frac{1}{8\pi M} = \frac{1}{4\pi R}$, for this observer each microscopic degree of freedom will attribute an energy $\frac{1}{2}k_B T = \frac{k_B}{8\pi R}$, note that a fundamental degree of freedom of the black hole will have the same energy if we divide M evenly over $N_{horizon} = A$, then, the total number microscopic degrees of freedom of the object at infinity is $N_{infinity} = \frac{8\pi ER}{k_B}$, and the entropy is $S_{infinity} = 2\pi ER$. On the other hand, when we place the object just hanging out the black hole, that is the proper distance from the center of the object to the horizon is r , the infinity observer will attribute an energy $\frac{Er}{2R} = \frac{Er}{4M}$ to the object, where $V = \frac{r}{2R}$ is the red shift factor, when it captured by the black hole, the entropy change of the black hole

is $\delta S_{BH} = 8\pi M \delta M = 2\pi E r$, then, the entropy of the object should be bounded by δS_{BH} , that is $S_{hanging} \leq 2\pi E r$. We found that for an infinity observer in Schwarzschild black hole background, the same object placed in different location that have different gravitational potential will be attributed different entropy. This means when an object is in a background of gravity, the object is not a isolate system, it's entropy is determined by the observer as well as the gravitational background.

Next, we want to show that the eq.(5) is consistent with the entropy bound proposed both by Bekenstein[24] and Susskind[23]. First, let's go back to the problem noticed by Verlinde in Sec. 6.4 of Ref.[4], which state that if one endow an Unruh temperature to a holographic screen that is not a horizon, the following entropy formula $S_{screen} = \frac{c^3}{4G\hbar} \int_S dA$, will violate the Bekenstein entropy bound. This problem can be clarified, as we state above different observers will attribute different amount of entropy to the same object, from the Rindler observer's point of view the object is no longer an isolate system in ordinary asymptotically flat spacetime, it is not a contradiction that a inertial observer who looks the object as a complete, weakly self-gravitating, isolate system in ordinary asymptotically flat spacetime will endow it a different amount of entropy.

Now, let's explain the consistency of entropic interpretation of gravity and entropy bound more explicit. Using eq.(5) and (2), we get

$$S = -k_B \frac{\Phi}{2c^2} \frac{Ac^3}{G\hbar} \leq \frac{Ak_B c^3}{4G\hbar}, \quad (7)$$

we use the fact that the maximum value of the ratio $-\Phi/2c^2$ is $1/4$ when the maximum coarse graining happens at horizons. This is just Susskind's holographic entropy bound, and we should also note that this is from the point of view of the observers that looks the object as a complete, weakly self-gravitating, isolate system in ordinary asymptotically flat spacetime.

If we use the equipartition rule $E = \frac{1}{2} N k_B T$, then, eq.(5) can be write as:

$$S = -k_B \frac{\Phi}{2c^2} \frac{2E}{k_b T}. \quad (8)$$

Due to the Unruh effect, for the system with mass $M = E/c^2$ which can be surrounded by a sphere of radius R , it is argue that $T \geq \frac{1}{8\pi M} \geq \frac{1}{4\pi R}$ [31]. The Bekenstein entropy bound $S \leq 2\pi E R$ is followed straightforward.

V. DISCUSSIONS AND CONCLUSIONS

The entropic interpretation of gravity present in Verlinde's paper is a very provocative idea, this is a good start, but the theory is also very incomplete, first of all, these idea should be recast in more precise way, and we still have a long journey to go to build a complete gravity theory.

In this small note, we present some explanations to some essential conceptual aspects that will fill some logical gap in interpreting the gravity as an entropic force which is missed in Verlinde's origin paper, and we think this is a small step to get a more precise theory. We have argued that thermodynamic quantities are observer dependent is very important concept in studying spacetime thermodynamic. In curved spacetime, energy is a observer dependent quantity, and particles also become an observer dependent notion, observers can disagree on the entropy and temperature of a system. Horizon is observer dependent and the thermodynamic laws established in spacetime thermodynamic are crucial related to it, because the entropy and temperature can not be defined without horizon, moreover, horizon plays as a edge of a gravitational thermodynamic system. The entropic interpretation of gravity give us some clue to understand that coarse graining will leave informations in the gravitational potential, this message can help us to link different observer's point of views about the same object.

From the point view of Verlinde, gravitational force is caused by entropy gradients when locations of material bodies changes, which means, gravity is a statistical tendency to return to a maximal entropy state. We know every object carries a amount of entropy, so the process in tending to the maximal entropy state is a process to rearrange the information, this is closely related the maximum speed of process information for a system. We will investigate the relation between the entropic interpretation of gravity and Margolus-Levitin Theorem in future.

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